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Fundamental Aspects in the Quantitative Ultrasonic Determination of Fracture Toughness: General Equations

L. S. Fu

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Fundamental Aspects in the Quantitative Ultrasonic Determination of Fracture Toughness: General Equations

L. S. Fu
The Ohio State University
Columbus, Ohio

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INTRODUCTION

Recent experimental and theoretical studies related to ultrasonics have demonstrated that the scattering of elastic waves by defects in a material provides information about both. This information may be used for determining the properties of any imperfection such as the size and orientation of a crack or it may be used for determining the mechanical and strength properties of the material.

Vary and his associates have emphasized the aspects of applying ultrasonics to material evaluation [1,2,3]. Their material information contents are experimentally measured in terms of ultrasonic attenuation and velocity factors. Using these concepts and a simple model, Vary [4] presented some very useful empirical relations that correlate these factors to the fracture toughness and the yield stress and gave data for two maraging steels and a titanium alloy.

The above mentioned findings are indeed very interesting and yet somewhat intriguing. After going through an extensive literature review, the author [5] suspected that the links between the strength properties (fracture toughness and yield stress) and the ultrasonic factors (attenuation and velocity factors) are the material microstructural parameters. It was pointed out that the size of the second phase particles and the distance between them play a very important role in material resistance to fracture.

In a first step toward defining the relation between K_{Ic} , fracture toughness and $v_L \beta_\delta / m$, the ultrasonic attenuation factor, the interaction of a pair of second phase particles (referred to as inhomogeneities) is studied. Since energy is trapped or dissipated in the vicinity of the

neighboring inhomogeneities, the ultrasonic factors measured at far field reflect this energy loss. Assuming that the attenuation is a function of incident wave frequency and that the energy trapped in the vicinity of the inhomogeneities provides the fracture energy a relation between the fracture toughness and the ultrasonic attenuation factors can be obtained. To accomplish this purpose both the far field and the near field solutions of the interaction problem are needed.

Currently, scattering theory of a single flaw is available [5]. The approach of Gubernatis, Domany and Krumhansl [6] is of particular interest in that they gave the scattered field solution far from the flaw in terms of the displacements and strains in the scatterer, i.e. the inhomogeneity. They also expressed the physical quantities such as total and differential cross sections in terms of the scattered field quantities. Since the strains and displacements inside the scatterer are not available for the dynamic case, they used the results from Eshelby [7] for the static case and studied the case appropriate to long wave limit [8].

The purpose of this report is to study the dynamic response of inhomogeneities, one or two, and to determine the strains and displacements inside them under incident plane waves as depicted in Figs. 1 and 2. The method of equivalent inclusion is used [9]. The underlining approach, the formulation and governing equation for the eigenstrains, and the determination of the energy due to the presence of the inhomogeneities are presented in this report. The derivation of the correlation between K_{1c} and $v_L \beta_\delta$ will follow later.

REVIEW OF SCATTERING THEORY: SINGLE FLAW

In a recent article, Gubernatis, Domany and Krumhansl (GDK) [6] summarized their recent work on elastic wave scattering with application to nondestructible evaluation. They studied the elastic scattering of a single flaw (be it a void, a crack or an inhomogeneity) under an experimental situation as depicted in Fig. 1. The scattered amplitudes and cross sections (measurable quantities) were derived specifically in terms of the scattered fields of stress and displacement at large distance from the scatterer.

For an incident plane wave of angular frequency ω and wave vector $\bar{k}$, Fig. 1, the incident displacement vector is

$$\overset{\circ}{u}_i(\bar{r},t) = \overset{\circ}{u}_i \exp i(\bar{k} \cdot \bar{r} - \omega t) \tag{1}$$

where

$$|\bar{k}| = \alpha = \omega/v_L$$

$$\beta = \omega/v_T$$

v_L = longitudinal sound velocity

v_T = transverse sound velocity

Employing integral theorems and the Green's function approach, they showed that the final form of the basic scattering equation is

$$u_i(\bar{r}) = \overset{\circ}{u}_i(\bar{r}) + \overset{s}{u}_i(\bar{r}) \tag{2}$$

in which

$$\overset{s}{u}_i(\bar{r}) = \delta\rho\, \omega^2 \int_{R_2} dv'\, g_{im}(\bar{r} - \bar{r}')u_m(\bar{r}') + \delta\, c_{jklm} \int_{R_2} dv'\, g_{ij,k}(\bar{r} - \bar{r}')\, u_{l,m}(\bar{r}') \tag{3}$$

where

$\delta\rho = \rho_{flaw} - \rho_{matrix}$

$\delta\, c_{jklm} = (c_{jklm})_{flaw} - (c_{jklm})_{matrix}$

R_2 = region occupied by scatter

For an isotropic elastic medium, in the far field at a distance r from the defect, they found that the scattered field depended on a certain vector, the f-vector, as follows:

$$u_i^s \sim e_i e_j f_j(\,\bar{\alpha}\,)\,\frac{e^{i\alpha r}}{r} + \\ + (\delta_{ij} - e_i e_j)\; f_j(\,\bar{\beta}\,)\,\frac{e^{i\beta r}}{r}\quad,\; r \to \infty \tag{4}$$

where the f-vector is defined as

$$f_j(\bar{k}) = \frac{k^2}{4\pi\rho\omega^2}\;[\delta\rho\;\omega^2 \int_{R_2} dv\; u_i\; \exp(-i\bar{k}\cdot\bar{r}) \\ + i\;k\;e_j\;\delta\;c_{ijkl} \int_{R_2} dv\; \epsilon_{kl}\; \exp(-i\bar{k}\cdot\bar{r})] \tag{5}$$

and is dependent upon the differences in material density and properties, between the matrix and the defect, the incident wave field and the total displacement and strain fields "inside" the scatterer, R_2.

With the asymptotic value of $\overline{u^s}$ given in Eq. (4), the asymptotic value of σ_{ij}^s can be obtained as

$$\sigma_{ij}^s \sim i\lambda\alpha\,\frac{e^{i\alpha r}}{r}\;e_k\;\delta_{ij}\;f_k(\bar{\alpha}) \\ + i\mu\;[2\alpha\,\frac{e^{i\alpha r}}{r}\;[e_i e_j e_k f_k(\bar{\alpha})] \\ + \beta\,\frac{e^{i\beta r}}{r}\;[e_i f_j(\bar{\beta}) + e_j f_i(\bar{\beta})] \\ - 2\beta\,\frac{e^{i\beta r}}{r}\;e_i e_j e_k f_k(\bar{\beta})\;\}\qquad r \to \infty \tag{6}$$

and various cross section can be expressed in terms of the scattered asymptotic values. For example, the differential cross section, for any given frequency it is a measure of the fraction of incident power scattered into a particular direction, is found to be

$$\frac{dP(\omega)}{d\Omega} = \lim_{r\to\infty} \frac{\langle r^2 e_i \sigma_{ij}^S \dot{u}_j^S \rangle}{\langle I^\circ \rangle} \tag{7}$$

where a dot denotes time differentiation and an angle bracket denotes time averaging, i.e.

$$\langle f(t) \rangle = \frac{1}{T} \cdot \int_0^T f(t)dt$$

The differential $d\Omega$ is the differential element of a solid angle. The total cross section is simply

$$P(\omega) = \int_{4\pi} d\Omega \frac{dP(\omega)}{d\Omega} \tag{8}$$

From Eqs. (4-8) it can be seen that the far field solution, the scattered amplitudes and cross sections, depends upon the determination of the displacement and strain fields inside the scatterer and the evaluation of the volume integrals in Eq. (5), see statements following Eq. 5, Ref. [9]. The displacement and strain fields are currently not available for the dynamic case. As a remedy, Gubernatis [8] used the results obtained for the static case from Eshelby [7], obtained by employing a method called equivalent inclusion method.

INHOMOGENEITIES IN A TIME-HARMONIC WAVE FIELD

From the above formulation, it is clearly seen that displacement and strain fields inside scatterers (inhomogeneities) play a very important role in nondestructive evaluation. In studying the interaction between two inhomogeneities, the dynamic version of the equivalent inclusion method is used. The method was first employed by Mura [10] and his associates [11] in studying composites.

Let the ultrasonic experimental situation be depicted as in Fig. 2 such that the incident power is along the positive z-axis. Let the total strain be the combination of elastic strain and non-elastic (or eigen-) strain:

$$\epsilon_{rs} = \epsilon_{rs}^{e} + \epsilon_{rs}^{*} \tag{9}$$

where ϵ_{rs}, ϵ_{rs}^{e} and ϵ_{rs}^{*} are the total strain, elastic strain and the eigenstrain, respectively. The eigenstrain is the non-elastic strain which is caused by a change of the form of an inclusion, which if the surrounding matrix material were absent, would have gone through some homogeneous deformation. Due to the difference in material properties between the matrix and the inclusions, this change of form causes disturbance in stress and associated strain in the material.

The equations of motion for a continuum are

$$\sigma_{jk,k} = \rho \ddot{u}_j \tag{10}$$

where a dot indicates a differentiation with respect to time while a subscript comma indicates a spatial differentiation. In a linear elastic matrix with small strain deformation, the strain-displacement relations are

$$\epsilon_{rs} = \frac{1}{2}(u_{r,s} + u_{s,r}) \tag{11}$$

and the generalized Hooke's law is

$$\sigma_{jk} = c_{jkrs}\,\epsilon^{e}_{rs} \tag{12}$$

where c_{jkrs} are the elastic constants. Employing Eqs. (9-12) the displacement equations of motion can be written as:

$$c_{jkrs}\,u_{r,sk} - \rho\,\ddot{u}_j = c_{jkrs}\,\epsilon^{*}_{rs,k} \tag{13}$$

If the matrix is isotropic the elastic constants c_{jkrs} can be expressed as

$$c_{jkrs} = \lambda\,\delta_{jk}\delta_{rs} + \mu\,\delta_{jr}\delta_{ks} + \mu\,\delta_{js}\delta_{kr} \tag{14}$$

where λ, μ are Lame's constants and δ_{ij} is the Kronecker's delta.

If the displacement and strain fields are time-harmonic, they can be written as

$$u_j(\bar{r},t) = u_j(\bar{r})\,\exp(-i\omega t) \tag{15}$$

$$\epsilon^{*}_{rs}(\bar{r},t) = \epsilon^{*}_{rs}(\bar{r})\,\exp(-i\omega t) \tag{16}$$

where ω is the frequency of the incident wave and $i \cdot i = -1$. A substitution of Eqs. (15,16) in Eq. (13) leads to

$$c_{jkrs}\,u_{r,sk} + \rho\omega^2 u_j = c_{jkrs}\,\epsilon^{*}_{rs,k} \quad \text{in } v \tag{17}$$

For a body of volume v and surface s containing an arbitrary distribution of eigenstrain, the traction free boundary condition can be expressed as

$$c_{jkrs}\,u_{r,s}\,n_k = c_{jkrs}\,\epsilon^{*}_{rs}\,n_k \quad \text{on } s \tag{18}$$

Consider now the associated Green's function with homogeneous boundary conditions:

$$c_{jkrs}\, g_{rm,sk} - \rho\, \ddot{g}_{jm} = -\delta_{jm}\, \delta(\bar{r}-\bar{r}',t), \quad \text{in } v \tag{19}$$

$$c_{jkrs}\, g_{rm,s}\, n_k = 0\,, \quad \text{on } s \tag{20}$$

Here, the Green's function $g_{jm}(\bar{r}-\bar{r}',t)$ represents the displacement in the j-direction at point $\bar{r}$ by a unit body force in the m-direction applied at the point $\bar{r}'$. The points defined by $\bar{r}$ and $\bar{r}'$ are referred to the observation and source points, respectively. The Dirac delta function, $\delta(\bar{r}-\bar{r}',t)$, represents the body force. For the time-harmonic case

$$\delta(\bar{r}-\bar{r}',t) = \delta(\bar{r}-\bar{r}')\exp(-i\omega t) \tag{21}$$

$$g_{jm}(\bar{r}-\bar{r}',t) = g_{jm}(\bar{r}-\bar{r}')\exp(-i\omega t) \tag{22}$$

The substitution of the above equation in Eqs. (19,20) yields

$$c_{jkrs}\, g_{rm,sk} + \rho\omega^2\, g_{jm} = -\delta_{jm}\, \delta(\bar{r}-\bar{r}')\,, \quad \text{in } v \tag{23}$$

$$c_{jkrs}\, g_{rm,s}\, n_k = 0\,, \quad \text{on } s \tag{24}$$

To derive an integral representation for the solution to the eigenstrain problem, a dynamic version of the Betti-Rayleigh reciprocal theorem is considered. Multiplying Eq. (17) by g_{jm} and Eq. (23) by u_j and substituting one from the other the following is obtained:

$$u_m(\bar{r}') = \int_v c_{jkrs}\,(g_{jm}\, u_{r,sk} - u_j\, g_{rm,sk} - g_{jm}\, e^{*}_{rs,k})dv \tag{25}$$

after an integration over the volume v. Noting the symmetric properties of c_{jkrs} and applying Gauss' divergence theorem and boundary conditions, an integral expression of the displacement field in terms of the eigen-

strains is obtained as follows:

$$u_m(\bar{r}') = \int_s c_{jkrs}\, \epsilon^*_{rs}\, g_{jm}\, n_k\, ds$$

$$-\int_v c_{jkrs}\, g_{jm}\, \epsilon^*_{rs,k}\, dv \qquad (26)$$

With the application of Gauss' theorem once more, the above equation can be reduced to a more compact form:

$$u_m(\bar{r}') = \int_v c_{jkrs}\, g_{jm,k}\,(\bar{r}-\bar{r}')\, \epsilon^*_{rs}(\bar{r})dv \qquad (27a)$$

or

$$u_m(\bar{r}) = -\int_v c_{jkrs}\, g_{jm,k}\,(\bar{r}-\bar{r}')\, \epsilon^*_{rs}(\bar{r}')dv' \qquad (27b)$$

where

$$g_{jm,k} = \frac{\partial}{\partial x_k}\, g_{jm} = -\frac{\partial}{\partial x_k'}\, g_{jm} \qquad (28)$$

ISOTROPIC LINEAR ELASTIC MEDIUM

For a linear elastic, isotropic and infinitely extended material, the Green's function is well-known:

$$g_{jm}(\bar{r}-\bar{r}') = \frac{1}{4\pi\rho\omega^2} \{ \beta^2 \frac{\exp i\beta R}{R} \delta_{jm}$$

$$- \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_m} [\frac{\exp i\alpha R}{R} - \frac{\exp i\beta R}{R}] \} \tag{29}$$

where

$$R = |\bar{r}-\bar{r}'|$$

$$\alpha^2 = \frac{\rho\omega^2}{\lambda+2\mu} = \frac{\omega^2}{V_L^2}$$

$$\beta^2 = \frac{\rho\omega^2}{\mu} = \frac{\omega^2}{V_T^2}$$

By substituting Eq. (29) in Eq. (27b) the induced displacement can be shown to be

$$u_m(\bar{r}) = - \frac{1}{4\pi\rho\omega^2} [\alpha^2\lambda \Psi_{rr,m}$$

$$+ 2\mu \beta^2 \Phi_{mk,k} - 2\mu \Psi_{jk,jkm}$$

$$+ 2\mu \Phi_{jk,jkm}] \tag{30}$$

where

$$\Psi_{ij} = \iiint_\Omega \epsilon^*_{ij}(\bar{r}') \frac{\exp(i\alpha R)}{R} d\bar{r}' \tag{31}$$

$$\Phi_{ij} = \iiint_\Omega \epsilon^*_{ij}(\bar{r}') \frac{\exp(i\beta R)}{R} d\bar{r}' \tag{32}$$

From Eq. (30) the strain field can be obtained by direct differentiation as follows:

$$\epsilon_{mn}(\bar{r}) = -\frac{1}{4\pi \ldots}\,[\ldots\,\psi_{rr,mn}$$

$$+ \ldots\,(\phi_{mk,kn} + \phi_{nk,km})$$

$$- 2\mu\,\psi_{jk,kjmn} + 2\mu\,\phi_{jk,kjmn}\,] \qquad (33)$$

The displacement and strain field given in Eqs. (30,33) are those due to the presence of eigenstrains in the regions of inhomogeneities Ω's, respectively. They obviously depend on the form of the eigenstrains. Since the eigenstrains are not a known priori, it is convenient to expand the eigenstrain in a form of polynomial [12]

$$\epsilon^{*}_{ij} = B_{ij} + B_{ijk}\,x_k + B_{ijkl}\,x_k\,x_l + \ldots \qquad (34)$$

in the region where the eigenstrains are present. The quantities B_{ij}, B_{ijk}, ... are constants symmetric with respect to the free indices i and j and having values independent of the order in which the summation indices appear, i.e. $B_{ijkl} = B_{ijlk}$, $B_{ijklm} = B_{ijkml}$, etc. Using Eq. (34) for e^{*}_{ij} and substituting it in Eqs. (31,32), the functions Ψ_{ij} and Φ_{ij} are found to be in terms of the constants B_{ij}, B_{ijk}, ... and some volume integrals as follows:

$$\Psi_{ij} = B_{ij}\,\psi(\bar{r}) + B_{ijk}\,\psi_k(\bar{r}) + B_{ijkl}\,\psi_{kl}(\bar{r}) + \ldots \qquad (35)$$

$$\Phi_{ij} = B_{ij}\,\phi(\bar{r}) + B_{ijk}\,\phi_k(\bar{r}) + B_{ijkl}\,\phi_{kl}(\bar{r}) + \ldots \qquad (36)$$

where

$$\psi(\bar{r}) = \iiint_{\Omega} \frac{\exp(i\alpha R)}{R}\,dv'$$

$$\psi_k(\bar{r}) = \iiint_{\Omega} x'_k\,\frac{\exp(i\alpha R)}{R}\,dv'$$

....

$$\psi_{kl\ldots s}(\bar{r}) = \iiint_{\Omega} x'_k\, x'_l \ldots x'_s \frac{\exp(i\alpha R)}{R}\, dv' \qquad (37)\ \text{(cont'd)}$$

$$\phi(\bar{r}) = \iiint_{\Omega} \frac{\exp(i\beta R)}{R}\, dv'$$

$$\phi_k(\bar{r}) = \iiint_{\Omega} x'_k \frac{\exp(i\beta R)}{R}\, dv'$$

....

$$\phi_{kl\ldots s}(\bar{r}) = \iiint_{\Omega} x'_k\, x'_l \ldots x'_s \frac{\exp(i\beta R)}{R}\, dv'$$

The substitution of Eqs. (35,36) in Eq. (33) leads to

$$\varepsilon_{mn}(\bar{r}) = D_{mnkj}(\bar{r})\, B_{kj} + D_{mnkjl}(\bar{r})\, B_{kjl} + \ldots \qquad (38)$$

where $B_{kj\ldots s}$ are constants, and

$$\begin{aligned} 4\pi\rho\omega^2 D_{mnkj}(\bar{r}) &= 2\mu(\psi_{,kjmn} - \phi_{kjmn}) \\ &\quad - \mu\, \beta^2(\phi_{,kn}\, \delta_{jm} + \phi_{,mk}\, \delta_{jn}) \\ &\quad - \lambda(\alpha^2 \psi_{,mn}\, \delta_{kj}) \qquad (39) \\ 4\pi\rho\omega^2 D_{mnkjl}(\bar{r}) &= 2\mu(\psi_{l,kjmn} - \phi_{l,kjmn}) \\ &\quad - \mu\, \beta^2(\phi_{l,kn}\, \delta_{jm} + \phi_{l,mk}\, \delta_{jn}) \\ &\quad - \lambda\, \alpha^2\, \psi_{l,mn}\, \delta_{kj} \end{aligned}$$

....

It should be noted that the D_{kjmn} are symmetric with respect to k,j and m,n. Generally, $D_{ijkl\ldots}(\bar{r}) \neq D_{klij\ldots}(\bar{r})$ unless $i \neq j$ and $k \neq l$, e.g. $D_{1122} \neq D_{2211}$ but $D_{1223} = D_{2312}$, etc.

The development so far has reduced the determination of the displacement and strain field to the determination of the constants B_{ij}, $B_{ijk\ldots}$

and the volume integrals given in Eq. (37). For the static case, α , $\beta \to 0$, Eshelby showed that $D_{mnkj}(\bar{r})$ is a constant and $D_{mnkjl\ldots}$ are zero. Once the integrals in Eq. (37) are evaluated, the solution now will depend upon the determination of the constants $B_{ij}, B_{ijk}, \ldots$.

EVALUATION OF VOLUME INTEGRALS ϕ'S AND ψ'S

Let the volume integrals given in Eq. (37) be denoted by

$$I = \iiint \rho(\bar{r}') \cdot \frac{\exp i\zeta R}{R} dv' \tag{41}$$

where $\zeta = \alpha$ for the ψ-integrals and $\zeta = \beta$ for the ϕ-integrals, $\rho(\bar{r}')$ is of the form of $(x')^{\lambda}\ (y')^{\mu}\ (z')^{\nu}$, Ω is an interior region where $\rho(\bar{r}')$ is distributed, and $R = |\bar{r} - \bar{r}'|$. Employing suitable Taylor series expansion and the multinomial theorem, the I-integral can be written in a reasonably convergent series as

$$I(\bar{r})_{>} = \sum_{n=o}^{\infty} \sum_{l=o}^{n} \sum_{k=o}^{n-l} \frac{(-1)^n}{l!k!(n-l-k)!} \cdot \frac{\partial^n}{\partial x^l \partial y^k \partial z^{n-l-k}} \left(\frac{\exp i\zeta r}{r}\right) \cdot$$

$$\iiint_{\Omega} (x')^l (y')^k (z')^{n-l-k}\ \rho(x',y',z')dx'dy'dz'\ ,\ \bar{r} \text{ outside } \Omega \tag{42}$$

and

$$I(\bar{r})_{<} = \sum_{n=o}^{\infty} \sum_{l=o}^{n} \sum_{k=o}^{n-l} \frac{(-1)^n}{l!k!(n-l-k)!} \cdot x^l y^k z^{n-l-k} \cdot$$

$$\iiint_{\Omega} \rho(x',y',z') \frac{\partial^n}{\partial x'^l \partial y'^k \partial z'^{(n-l-k)}} \left(\frac{\exp i\zeta r'}{r'}\right) dx'dy'dz',$$

$$\bar{r} \text{ inside } \Omega \tag{43}$$

When Ω is an ellipsoidal region the integrals in $I_{>}$ and $I_{<}$ can be readily obtained by using results from Dyson [12], e.g.

$$\iiint_{\Omega} \frac{\sin\zeta r'}{r'} dv' = 4\pi a_1 a_2 a_3 \sum_{m=1}^{\infty} \frac{(-1)^{m-1}\ \zeta^{2m-1}\ a_k^{m-1}\ a_k^{m-1}}{(2m-1)!(2m-1)(2m+1)}$$

$$\iiint \frac{\partial^2}{\partial x_p' \partial x_p'} \left[\frac{\sin \zeta r'}{r'}\right] dv' = 4\pi a_1 a_2 a_3 \sum_{m=1}^{\infty} \frac{(-1)^{m-1} \zeta^{2m-1}}{(2m-1)!} \cdot$$

$$\left\{ \frac{(2m-2)(2m-4)}{(2m-3)(2m-5)} a_k^{m-3} a_k^{m-3} a_p + \frac{(2m-2)(3)}{(2m-3)(2m-1)} a_k^{m-2} a_k^{m-2} \right\}$$

...

$$\iiint \frac{\cos \zeta r'}{r'} dv' = \sum_{m=o}^{\infty} \frac{(-1)^m \zeta^{2m}}{2m!} L_{m,o}$$

where

$$L_{m,o} = \frac{\pi a_1 a_2 a_3}{2^{2m}(m+1)} \sum_{m_1, m_2, m_3} a^{2m_1} a^{2m_2} a^{2m_3} \frac{2m_1!\,2m_2!\,2m_3!}{m_1!\,m_2!\,m_3!}$$

$$\cdot \int_o^{\infty} \frac{\psi^m \, d\psi}{(a_1{}^2+\psi)^{m_1} (a_2{}^2+\psi)^{m_2} (a_3{}^2+\psi)^{m_3} \sqrt{Q}} ,$$

$$m_1 + m_2 + m_3 = m ,$$

$$Q = (a_1{}^2+\psi)(a_2{}^2+\psi)(a_3{}^2+\psi) ,$$

...

in which $(x/a_1)^2 + (y/a_2)^2 + (z/a_3)^2 = 1$.

DETERMINATION OF EIGENSTRAINS

Let the inhomogeneities be situated as shown in Fig. 2, where Ω_I and Ω_{II} are the regions occupied by the inhomogeneities. If the applied elastic fields are denoted by strains $\varepsilon^a_{ij}(\bar{r})$, stresses $\sigma^a_{ij}(\bar{r})$ and displacements $u^a_i(\bar{r})$ and if the self-equilibrated fields due to the presence of the inhomogeneities are denoted by $\varepsilon_{ij}(\bar{r})$, $\sigma_{ij}(\bar{r})$ and $u_j(\bar{r})$ the total fields due to the applied field and the inhomogeneities are then the sum of the two. It is noted here that the time-dependence is suppressed. The MEI can be used to determine the eigenstrains $\overset{*}{\varepsilon}{}^{I}_{ij}(\bar{r})$ in Ω_I and $\overset{*}{\varepsilon}{}^{II}_{ij}(\bar{r})$ in Ω_{II} as follows:

$$\Delta C^{I}_{ijkl}\,\varepsilon_{kl}(\bar{r}) + C^{o}_{ijkl}\,\overset{*}{\varepsilon}{}^{I}_{kl}(\bar{r}) = -\Delta C^{I}_{ijkl}\,\varepsilon^{a}_{kl}(\bar{r}) \text{ in } \Omega_I \tag{44}$$

$$\Delta C^{II}_{ijkl}\,\varepsilon_{kl}(\bar{r}) + C^{o}_{ijkl}\,\overset{*}{\varepsilon}{}^{II}_{kl}(\bar{r}) = -\Delta C^{II}_{ijkl}\,\varepsilon^{a}_{kl}(\bar{r}) \text{ in } \Omega_{II} \tag{45}$$

where

$$\Delta C^{I}_{ijkl} = C^{I}_{ijkl} - C^{o}_{ijkl} \tag{46}$$

$$\Delta C^{II}_{ijkl} = C^{I}_{ijkl} - C^{o}_{ijkl} \tag{47}$$

and C^{o}_{ijkl}, C^{I}_{ijkl}, C^{II}_{ijkl} are the elastic moduli tensor of the matrix, the first inhomogeneity Ω_I, and the second inhomogeneity Ω_{II}, respectively.

Consider now the expansion of the eigenstrains in polynomial form such that

$$\varepsilon^{a}_{ij}(\bar{r}) = E_{ij} + E_{ijk}\,x_k + E_{ijkl}\,x_k\,x_l + \ldots \tag{48}$$

$$\overset{*}{\varepsilon}{}^{I}_{ij}(\bar{r}) = B^{I}_{ij} + B^{I}_{ijk}\,x_k + B^{I}_{ijkl}\,x_k\,x_l + \ldots \text{ in } \Omega_I \tag{49}$$

$$\bar{\varepsilon}^{*\,II}_{ij}(\bar{r}) = B^{II}_{ij} + B^{II}_{ijk}\,\bar{x}_k + B^{II}_{ijkl}\,\bar{x}_k\,\bar{x}_l + \ldots \quad \text{in } \Omega_{II} \tag{50}$$

where the barred and unbarred quantities are measured from the two different coordinate systems as shown in Fig. 2. The position vectors are related by

$$x^p_i = x^{\bar{o}}_i + a_{ij}\,\bar{x}^p_j \tag{51}$$

$$\bar{x}^p_i = (x^p_j - x^{\bar{o}}_j)\,a_{ji} \tag{52}$$

where a_{ij} is the coordinate transformation matrix. It is easy to show that the unknown constants B^{I}_{ij}, B^{II}_{ij}, B^{I}_{ijk}, B^{II}_{ijk}, ... in Eqs. (49,50) satisfy the following simultaneous equations:

$$\begin{aligned}
&\Delta C^{I}_{stmn}\,\{[D^{I}_{mnij}[0]B^{I}_{ij} + D^{I}_{mnijk}[0]B^{I}_{ijk} \\
&\quad + D^{I}_{mnijkl}[0]B^{I}_{ijkl} + \ldots] + a_{mc}a_{nh}\,[D^{II}_{chij}[0]B^{II}_{ij} \\
&\quad + D^{II}_{chijk}[0]B^{II}_{ijk} + D^{II}_{chijkl}[0]B^{II}_{ijkl} + \ldots]\} \\
&\quad + C^{o}_{stmn}\,B^{I}_{mn} = -\Delta C^{I}_{stmn}E_{mn}
\end{aligned} \tag{53}$$

$$\begin{aligned}
&\Delta C^{I}_{stmn}\,\{[\frac{\partial}{\partial x_p}\,D^{I}_{mnij}[0]B^{I}_{ij} + \frac{\partial}{\partial x_p}\,D^{I}_{mnijk}[0]B^{I}_{ijk} \\
&\quad + \frac{\partial}{\partial x_p}\,D^{I}_{mnijkl}[0]B^{I}_{ijkl} + \ldots] \\
&\quad + a_{mc}\,a_{nh}\,a_{pf}\,[\frac{\partial}{\partial \bar{x}_f}\,D^{II}_{chij}[0]B^{II}_{ij} \\
&\quad + \frac{\partial}{\partial \bar{x}_f}\,D^{II}_{chijk}[0]B^{II}_{ijk} + \frac{\partial}{\partial \bar{x}_f}\,D^{II}_{chijkl}[0]B^{II}_{ijkl} + \ldots\ ]\} \\
&\quad - C^{o}_{stmn}\,B^{I}_{mnp} = -\Delta C^{I}_{stmn}\,E^{I}_{mnp}
\end{aligned} \tag{54}$$

$$\frac{1}{2!}\,\Delta C^{I}_{stmn}\,\{[\frac{\partial^2}{\partial x_p \partial x_q}\,D^{I}_{mnij}[0]B^{I}_{ij} + \frac{\partial^2}{\partial x_p \partial x_q}\,D^{I}_{mnijk}[0]B^{I}_{ijk}$$

$$+ \frac{\partial^2}{\partial x_p \partial x_q}\,D^{I}_{mnijkl}[0]B^{I}_{ijkl} + \ldots\,]$$

$$+ a_{mc}\,a_{nh}\,a_{pf}\,a_{qg}\,[\frac{\partial^2}{\partial \bar{x}_f \partial \bar{x}_g}\,D^{II}_{chij}[0]B^{II}_{ij}$$

$$+ \frac{\partial^2}{\partial \bar{x}_f \partial \bar{x}_g}\,D^{II}_{chijk}[0]B^{II}_{ijk}$$

$$+ \frac{\partial^2}{\partial \bar{x}_f \partial \bar{x}_g}\,D^{II}_{chijkl}[0]B^{II}_{ijkl} + \ldots\,]\}$$

$$+ C^{o}_{stmn}\,B^{I}_{mnpq} = -\Delta C^{I}_{stmn}\,E_{mnpq} \quad , \qquad (55)$$

...

$$\Delta C^{II}_{stmn}\,\{a_{cm}a_{hn}\,[D^{I}_{chij}[\bar{0}]B^{I}_{ij} + D_{chijk}[\bar{0}]B^{I}_{ijk}$$

$$+ D^{I}_{chijkl}[\bar{0}]B^{I}_{ijkl} + \ldots] + [D^{II}_{mnij}[\bar{0}]B^{II}_{ij}$$

$$+ D^{II}_{mnijk}[\bar{0}]B^{II}_{ijk} + D^{II}_{mnijkl}[\bar{0}]B^{II}_{ijkl} + \ldots]\}$$

$$+ C^{o}_{stmn}\,B^{II}_{mn} = -\Delta C^{II}_{stmn}\,\bar{E}_{mn} \qquad (56)$$

$$\Delta C^{II}_{stmn}\,\{a_{cm}a_{hn}a_{fp}\,[\frac{\partial}{\partial x_f}\,D^{I}_{chij}[\bar{0}]B^{I}_{ij}$$

$$+ \frac{\partial}{\partial x_f}\,D^{I}_{chijk}[\bar{0}]B^{I}_{ijk} + \frac{\partial}{\partial x_f}\,D^{I}_{chijkl}[\bar{0}]B^{I}_{ijkl} + \ldots]$$

$$+[\frac{\partial}{\partial \bar{x}_p}\,D^{II}_{mnij}[\bar{0}]B^{II}_{ij} + \frac{\partial}{\partial \bar{x}_p}\,D^{II}_{mnijk}[\bar{0}]B^{II}_{ijk}$$

$$+ \frac{\partial}{\partial \bar{x}_p}\,D^{II}_{mnijkl}[\bar{0}]B^{II}_{ijkl} + \ldots\,]\}$$

$$+ C^{o}_{stmn}\,B^{II}_{mnp} = -\,C^{II}_{stmn}\,\bar{E}_{mnp} \qquad (57)$$

$$\frac{1}{2!}\Delta C^{II}_{stmn}\{a_{cm}a_{hn}a_{fp}a_{gq}[\frac{\partial^2}{\partial x_f\partial x_g}D^{I}_{chij}[\overline{0}]B^{I}_{ij}$$

$$+\frac{\partial^2}{\partial x_f\partial x_g}D^{I}_{chijk}[\overline{0}]B^{I}_{ijk}+\frac{\partial^2}{\partial x_f\partial x_g}D^{I}_{chijkl}[\overline{0}]B^{I}_{ijkl}+\ldots]$$

$$+[\frac{\partial^2}{\partial x_p\partial x_g}D^{II}_{mnij}[\overline{0}]B^{II}_{ij}+\frac{\partial^2}{\partial x_p\partial x_q}D^{II}_{mnijkl}[\overline{0}]B^{II}_{ijkl}+\ldots]\}$$

$$+C^{o}_{stmn}B^{II}_{mnpq}=-\Delta C^{II}_{stmn}\overline{E}_{mnpq}\quad. \tag{58}$$

etc.

in which the right hand side are determined by expanding the applied in a polynomial as in Eq. (48) and the equivalency equations used for any point $\overline{P}$ are

$$\Delta C^{I}_{ijkl}\{\varepsilon^{I}_{kl}(\vec{r}^{p})+\varepsilon^{II}_{kl}(\vec{r}^{p})\}+C^{o}_{ijkl}e^{*I}_{kl}(\vec{r}^{p})$$

$$=-\Delta C_{ijkl}\,\varepsilon_{kl}(r^{p})\ ;\ P\text{ in }\Omega_I \tag{59}$$

$$\Delta C^{II}_{ijkl}\{\varepsilon^{II}_{kl}(\vec{r}^{p})+\overline{\varepsilon}^{II}_{kl}(\vec{r}^{p})\}+C^{o}_{ijkl}\varepsilon^{*II}_{kl}(\vec{r}^{p})$$

$$=-\Delta C^{II}_{ijkl}\,\overline{\varepsilon}^{a}_{kl}(\vec{r}^{p})\ ;\ P\text{ in }\Omega_2 \tag{60}$$

The notation $D_{ijkl}[0]$, $\frac{\partial}{\partial x_p}D_{ijkl}[0]$ ... mean that the D's are evaluated at the point "O". The D's are defined in Eq. (39,40) ... . The superscripts I and II are referred to the regions Ω_I and Ω_{II} occupied by the inhomogeneities.

INTERACTION ENERGY

If an elastic body is subjected to surface tractions $t_i(\bar{r},t)$ and body forces $f_i(\bar{r},t)$, the induced elastic fields depend upon the elastic moduli of the body. Let $\sigma_{ij}^{in}(\bar{r},t)$, $\epsilon_{ij}^{in}(\bar{r},t)$, $u_i^{in}(\bar{r},t)$ and $\sigma_{ij}^{f}(\bar{r},t)$, $\epsilon_{ij}^{f}(\bar{r},\dot{t})$, $u_i^{f}(\bar{r},t)$ be the stress, strain and displacement fields induced when the elastic moduli are $C_{ijkl}^{in}(\bar{r})$ and $C_{ijkl}^{f}(\bar{r})$, respectively. The initial state, denoted by superscript <u>in</u> , can be considered as the state where there are no inhomogeneities and the final state, denoted by $\bar{f}$, can be considered as the state where there are inhomogeneities present.

Using the notations given in [14], the difference in power input and the rate of change in kinetic energy plus potential energy is

$$\Delta\dot{E} = \dot{K}^f - \dot{K}^{in} + \dot{U}^f - \dot{U}^{in}$$

$$- \int_V f_i(\dot{u}_i^{\,f} - \dot{u}_i^{in})dv - \int_S t_i(\dot{u}_i^{\,f} - \dot{u}_i^{in})ds \qquad (61)$$

where ΔE = interaction energy due to the presence of the inhomogeneities

$$K = \frac{1}{2}\int_V \rho\, \dot{u}_i\, \dot{u}_i\, dv = \text{kinetic energy}$$

$$U = \frac{1}{2}\int_V \sigma_{ij}\, \epsilon_{ij}\, dv = \text{strain energy}$$

Hence the interaction energy rate is

$$\dot{E}^f - \dot{E}^{in} = \int_V \rho(\ddot{u}_i^{\,f}\, \dot{u}_i^{\,f} - \ddot{u}_i^{in}\, \dot{u}_i^{in})dv$$

$$+ \frac{1}{2}\int_V (\sigma_{ij}^{f}\, \dot{u}_{i,j}^{f} - \sigma_{ij}^{in}\, \dot{u}_{i,j}^{in})dv$$

$$- \int_V f_i(\dot{u}_i^{\,f} - \dot{u}_i^{\,in})dv$$

$$- \int_S t_i(\dot{u}_i^{\,f} - \dot{u}_i^{\,in})ds \qquad (62)$$

The equations of motion for the induced fields are:

$$-f_i = \sigma_{ij,j}^{f} - \rho\ddot{u}_i^{f} = \sigma_{ij,j}^{in} - \rho\ddot{u}_i^{in} \tag{63}$$

$$t_i = \sigma_{ij}^{f} n_j = \sigma_{ij}^{in} n_j \tag{64}$$

Using integration by parts and the equations of motion plus boundary conditions, Eqs. (63,64), the following identity can be derived:

$$\iiint_V (\sigma_{ij}^{f} \dot{u}_{i,j}^{f} - \sigma_{ij}^{in} \dot{u}_{i,j}^{in})dv$$

$$= \iint_S (\sigma_{ij}^{f} \dot{u}_i^{f} - \sigma_{ij}^{in} \dot{u}_i^{in})n_j \, ds$$

$$- \iiint_V (\sigma_{ij,j}^{f} \dot{u}_i^{f} - \sigma_{ij,j}^{in} \dot{u}_i^{in})dv$$

$$= \iint_S t_i(\dot{u}_i^{f} - \dot{u}_i^{in})n_j \, ds$$

$$- \iiint_V \rho(\ddot{u}_i^{f} \dot{u}_i^{f} - \ddot{u}_i^{in} \dot{u}_i^{in})dv$$

$$+ \iiint_V f_i(\dot{u}_i^{f} - \dot{u}_i^{in})dv$$

$$= \iint_S (\sigma_{ij}^{in} \dot{u}_i^{f} - \sigma_{ij}^{f} \dot{u}_i^{in})n_j \, ds$$

$$- \iiint_V [(\sigma_{ij,j}^{in} - \rho\ddot{u}_i^{in})\dot{u}_i^{f} - (\sigma_{ij,j}^{f} - \rho\ddot{u}_i^{f})\dot{u}_i^{in}]dv$$

$$- \iiint_V \rho(\ddot{u}_i^{f} \dot{u}_i^{f} - \ddot{u}_i^{in} \dot{u}_i^{in})dv$$

$$= \iiint_V (\sigma_{ij}^{in} \dot{u}_{i,j}^{f} - \sigma_{ij}^{f} \dot{u}_{i,j}^{in})dv$$

$$+ \iiint_V \rho(\ddot{u}_i^{in} \dot{u}_i^{f} - \ddot{u}_i^{in} \dot{u}_i^{f})dv$$

$$- \iiint_V (\rho \ddot{u}_i^{f} u_i^{f} - \ddot{u}_i^{in} \dot{u}_i^{in})dv \tag{65}$$

The substitution of Eq. (65) in Eq. (62) leads to

$$
\begin{aligned}
\Delta\dot{E} &= -\frac{1}{2}\iint_S t_i(\dot{u}_i{}^f - \dot{u}_i{}^{in})ds - \frac{1}{2}\iiint_V f_i(\dot{u}_i{}^f - \dot{u}_i{}^{in})dv \\
&\quad + \frac{1}{2}\iiint \rho(\ddot{u}_i{}^f\,\dot{u}_i{}^f - \ddot{u}_i{}^{in}\,\dot{u}_i{}^{in})dv \\
&= -\frac{1}{2}\iint_S t_i(\dot{u}_i{}^f - \dot{u}_i{}^{in}) \\
&\quad - \frac{1}{2}\iiint_V [(f_i - \rho\ddot{u}_i{}^f)\dot{u}_i{}^f - (f_i - \rho\ddot{u}_i{}^{in})\dot{u}_i{}^{in}]\, dv \\
&= -\frac{1}{2}\iiint_V (\sigma_{ij}^{in}\,\dot{u}_{i,j}^{f} - \sigma_{ij}^{f}\,u_{i,j}^{in})dv \\
&= -\frac{1}{2}\iiint_V [\sigma_{ij}^{f}\,\dot{u}_{i,j}^{in} - \sigma_{ij}^{in}\,\dot{u}_{i,j}^{f}]dv
\end{aligned} \tag{66}
$$

The above equation may be expressed in terms of strains as follows:

$$
\begin{aligned}
\Delta\dot{E} &= -\frac{1}{2}\iiint_V (\sigma_{ij}^{f}\,\dot{\varepsilon}_{ij}^{f} - \sigma_{ij}^{in}\,\dot{\varepsilon}_{ij}^{in})dv \\
&= -\frac{1}{2}\iiint_V (\sigma_{ij}^{in}\,\dot{\varepsilon}_{ij}^{f} - \sigma_{ij}^{f}\,\dot{\varepsilon}_{ij}^{in})dv
\end{aligned} \tag{67}
$$

When inhomogeneities exist in the body, the following definitions are noted:

$$
\begin{aligned}
\dot{\varepsilon}_{ij}^{in} &= \dot{\varepsilon}_{ij}^{a}\,,\ \sigma_{ij}^{in} = \sigma_{ij}^{a} = C_{ijkl}^{o}\,\dot{\varepsilon}_{kl}^{a}\,, \\
\dot{\varepsilon}_{ij}^{f} &= \dot{\varepsilon}_{ij}^{a} + \dot{\varepsilon}_{ij}
\end{aligned} \tag{68}
$$

$$
\begin{aligned}
\sigma_{ij}^{f} &= C_{ijkl}^{o}\,(\varepsilon_{kl}^{a} + \varepsilon_{kl}) \text{ in matrix} \\
&= C_{ijkl}^{I}\,(\varepsilon_{kl}^{a} + \varepsilon_{kl}) = C_{ijkl}^{o}\,(\varepsilon_{kl}^{a} + \varepsilon_{kl} - \varepsilon_{kl}^{*I}) \text{ in } \Omega_I \\
&= C_{ijkl}^{II}\,(\varepsilon_{kl}^{a} + \varepsilon_{kl}) = C_{ijkl}^{o}\,(\varepsilon_{kl}^{a} + \varepsilon_{kl} - \varepsilon_{kl}^{*II}) \text{ in } \Omega_{II}
\end{aligned}
$$

Hence the interaction energy due to the presence of inhomogeneities is

$$
\Delta E = E^f - E^{in} = -\frac{1}{2}\iiint_\Omega C_{ijkl}^{o}\,\varepsilon_{kl}^{*}\,\varepsilon_{ij}^{a}\, dv
$$

where $\Omega = \Omega_I + \Omega_{II} + \ldots$ and ε_{kl}^{*} is the eigenstrain in each Ω .

CONCLUDING REMARKS

Quantitative determination of attenuation and velocity factors requires the solution of far fields and near fields for a material containing inhomogeneities under the excitation of incident power. The brief review of the existing scattering theory of a single flaw led to the realization of the importance of finding displacement and strain fields inside inhomogeneities. They are currently not available. Definition and general equations for time-harmonic displacement and strain fields in a pair of interacting inhomogeneities are given.

The interaction problems are presented via the dynamic eigenstrain concept. This approach converts the problem of dealing with inhomogeneous boundary conditions to that of dealing with an nonhomogeneous differential equation. The nonhomogeneous term is directly related to the strains and displacements in the scatterer and can be obtained by the method of equivalent inclusions.

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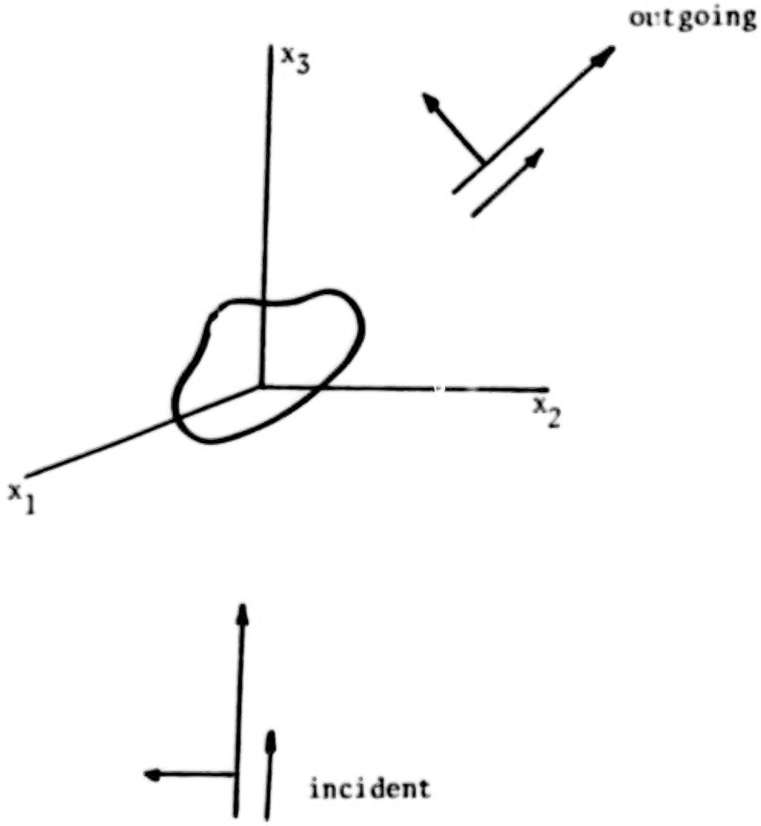

Fig. 1 The scattering geometry.

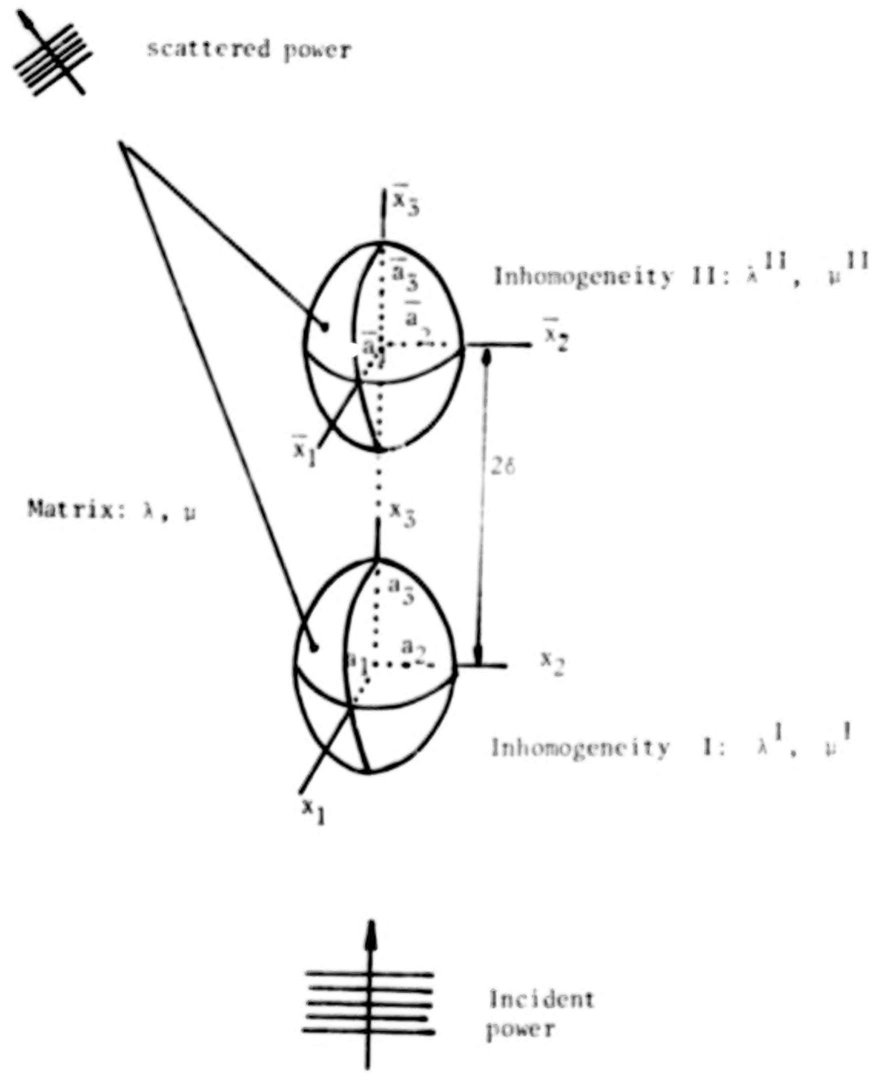

Fig. 2 Interaction of inhomogeneities with incident power along $+x_3$-axis.

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15. Supplementary Notes

Final report. Project Manager, Alex Vary, Materials Division, NASA Lewis Research Center, Cleveland, Ohio 44135.

16. Abstract

This report covers fundamental aspects of the problem of establishing a theoretical groundwork for experimentally-found correlations between ultrasonic and fracture toughness factors in polycrystalline metals. Recent experiments have shown that ultrasonic measurements of velocity and attenuation can be used to predict fracture toughness and associated material strength properties. The link between these material properties and ultrasonic factors are the microstructural parameters that interact with stress wave propagation during deformation and fracture. This study is concerned with the dynamic response of material inhomogeneities and the strains and displacements they undergo under incident stress waves. This study goes beyond the conventional static treatments and treats dynamic strains and displacements inside and outside scatterers. The underlying approach, the formulation and governing equations for the eigenstrains, and the determination of the energy due to the presence of inhomogeneities are presented in this report. The stress wave interaction problem is presented in terms of the dynamic eigenstrain concept.

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